Dynamical Cobordisms in String Theory

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Outline and references

Plan of the talk

- Review of Cobordism Conjecture
- Cobordism to Nothing in the Effective FT approach
- Local Analysis
- 10d Massive type IIA
- D-branes as Dynamical Cobordisms
- Conclusions

References

R.A.,J.Calderón-Infante, M. Delgado, J. Huertas, A. M. Uranga.
At the end of the world: Local Dynamical Cobordism.
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G.Buratti, M. Delgado, A.M. Uranga.
Dynamical tadpoles, stringy cobordism, and the SM from spontaneous compactification
J. High Energ. Phys. 06 (2021) 160 [preprint - arXiv 2104.02091]

J.Calderon-Infante, G.Buratti, M. Delgado, A.M. Uranga.
Dynamical cobordism and Swampland Distance Conjectures
J. High Energ. Phys. 10 (2021) 037 [preprint - arXiv 2107.09098]

J.McNamara, C. Vafa
Cobordism classes and the Swampland
[arXiv 1909.10355]

Let $\mathscr{T}_1$ and $\mathscr{T}_2$ be two D-dimensional QG theories. We say that they are equivalent, and we write:

$$\mathscr{T}_1 \sim_D^{QG} \mathscr{T}_2$$

if

- They correspond to two different regions of the same moduli space;
- The cost of energy to move from a region to the other is finite.

Quantum Gravity Cobordism Group

$$\Omega_D^{QG} = \{\text{D-dim QG theories}\}/ \sim_D^{QG}$$

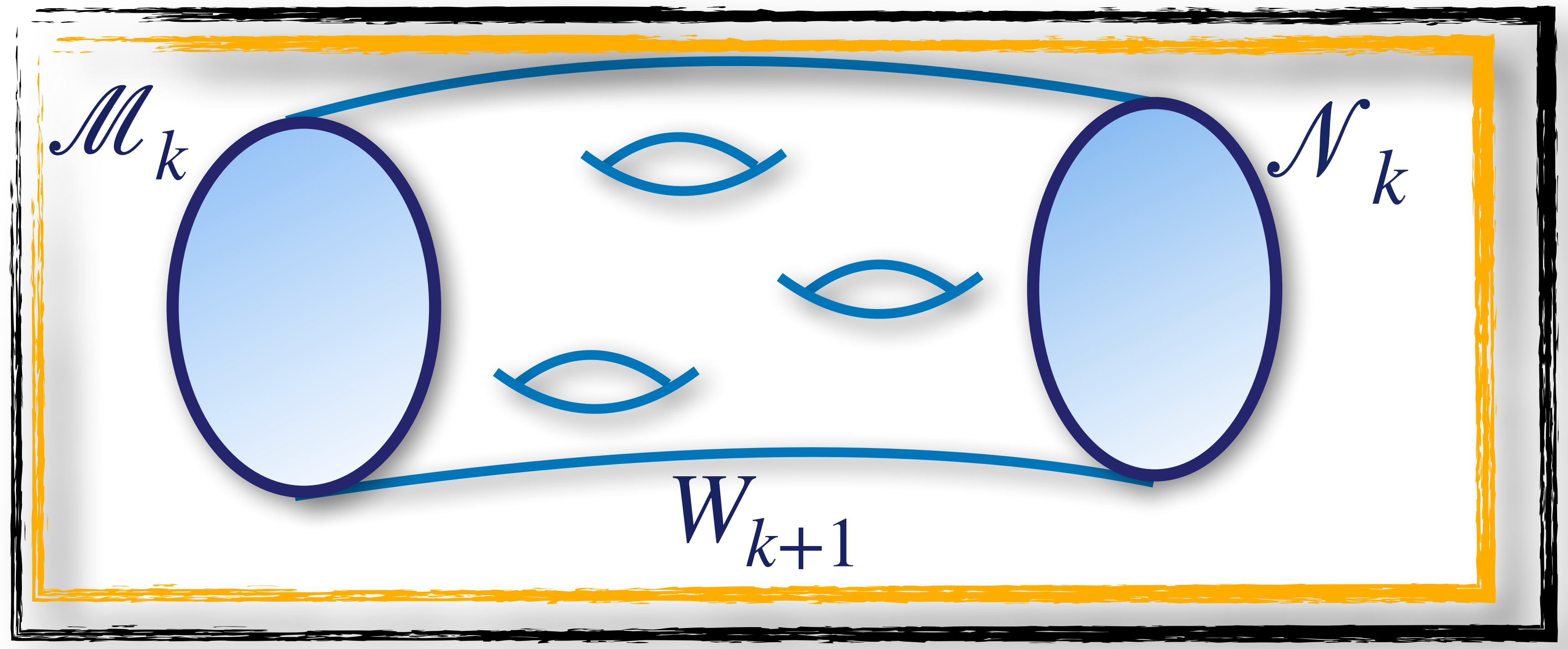
Geometric interpretation

- (D+k)-dim String Theories
- $\mathscr{M}_k$ smooth and compact

$$\mathscr{M}_k \sim_k \mathscr{N}_k$$

if

$$\mathscr{M}_k \sqcup \mathscr{N}_k = \partial W_{k+1}$$



Mathematical Cobordism Group

$$\Omega_k = \{\text{Compact,closed k-dim manifolds}\}/ \sim_k$$

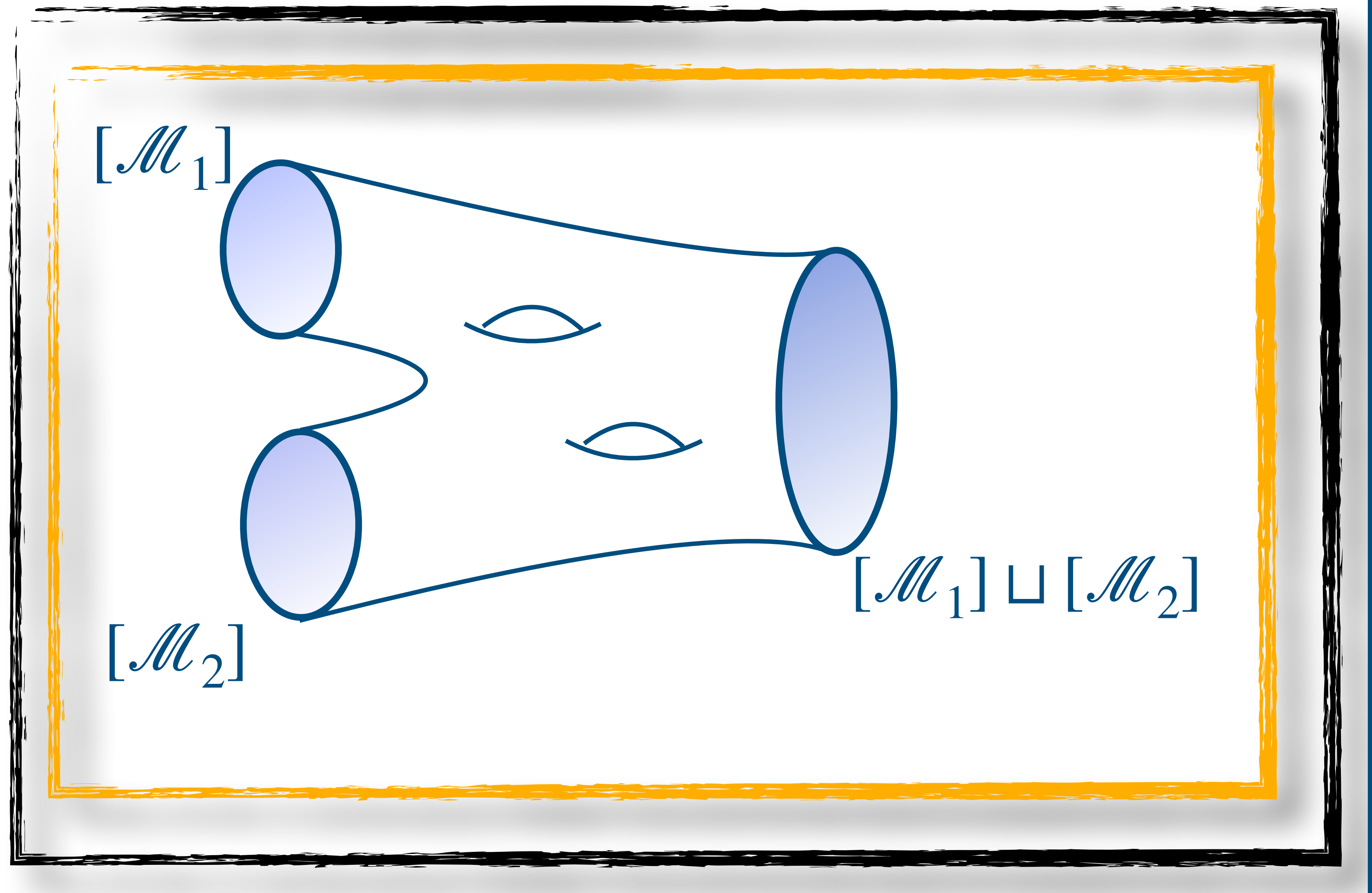
Group structure

The set of classes of equivalent backgrounds admits a group structure:

Composition law

$\forall [\mathscr{M}_1], [\mathscr{M}_2] \in \Omega_k$ there are a third element of the group defined through the disjoint union of two k-dim backgrounds

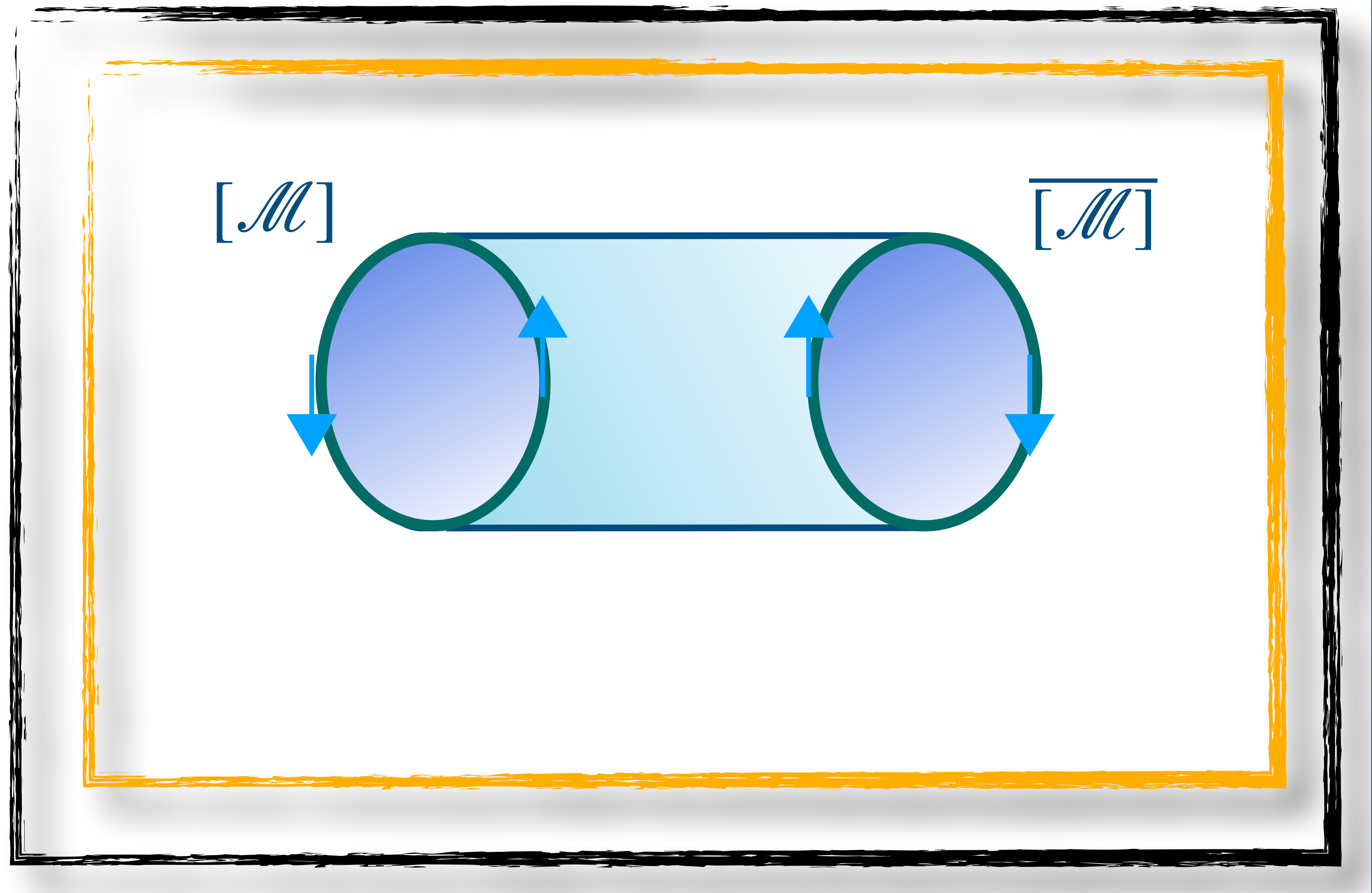
$$[\mathscr{M}_1] \sqcup [\mathscr{M}_2] = [\mathscr{M}_1 \sqcup \mathscr{M}_2]$$



Inverse element

For each element $[\mathscr{M}] \in \Omega_k$ there exist an element $\overline{[\mathscr{M}]} \in \Omega_k$, called the INVERSE, such that:

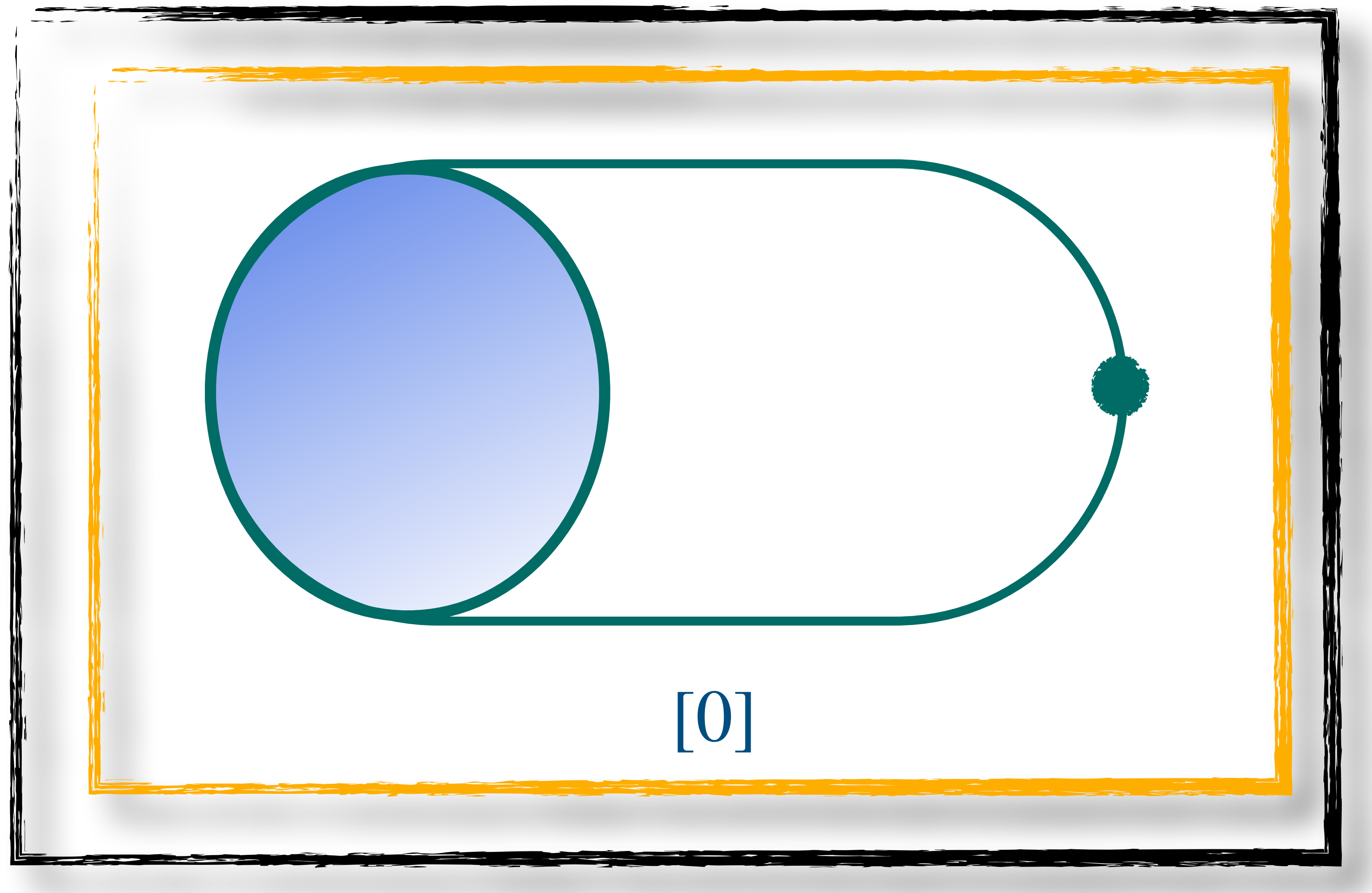
$$[\mathscr{M}] \sqcup \overline{[\mathscr{M}]} = [0]$$



Identity element

There is a class $[0] \in \Omega_k$ such that $[0] \sqcup [\mathscr{M}] = [\mathscr{M}] + [0] = [\mathscr{M}]$, for any $[\mathscr{M}] \in \Omega_k$.

Such class has to contain the empty background



Cobordism Conjecture

Backgrounds in different cobordism classes are identified by different values of a certain invariant quantity.
We identify this invariant as the charge respect to a certain Global Symmetry.
The abstence of Global Symmetries in Quantum Gravity ([Banks, Seiberg '10; Banks,Dixon '88]) implies the Cobordism Conjecture.

Cobordism Conjecture

Any consistent D-dimensional theory of Quantum Gravity is in the trivial cobordism class

$$\Omega_D^{QG} = [0]$$

[McNamara,Vafa '19]

Topological implications:

- For any pair of consistent D-dimensional QG theories there exists an allowed configuration connecting them.
- Any QG theory admits a configuration ending space-time at a codimension 1 boundary (*End of the World configurations*).

We want to study realization of *End of The World configurations* following an Effective Field theory approach in String Theory. We call *Cobordisms to Nothing* the configurations realizing this End of The-World.

How can we identify a Cobordism to Nothing looking at the Effective Field Theory?

(I) The EFT admits a R*unning Solution* along one spacetime direction

(II) The solution hits a *Singularity* at a F*inite* distance Δ in spacetime, at which the Ricci curvature diverges

(III) When we approach to the Singularity the distance $\mathscr{D}$ in moduli space goes to infinity.

We call ***End of The World*** brane (ETW) the source for such *Singularity.*

Local Analysis

- d-*dimensional* Einstein gravity coupled to a scalar with *generic* potential:

$$S = \int d^d x \sqrt{-g}\,[R - \frac{1}{2}(\partial\phi)^2 - V(\phi)]$$

- Ansatz solution:

$$ds^2 = e^{-2\sigma(y)} ds^2_{d-1} + dy^2$$
$$\phi = \phi(y)$$

- Equations of motion for flat *(d-1)-dimensional* slices:

$$\phi'' - (d-1)\sigma'\phi' - \partial_\phi V = 0$$
$$\frac{1}{2}(d-1)(d-2)\sigma'^2 + V - \frac{1}{2}\phi'^2 = 0$$
$$(d-2)\sigma'' - \phi'^2 = 0$$

- Imposing the condition that the scalar goes to infinity in a finite spacetime distance. Solving the EoMs…

We obtain a Local Universal Description in the near ETW-brane regime

Critical Exponent controlling the theory

$$\delta = 2\sqrt{\frac{d-1}{d-2}(1-a)}$$

- An exponential potential dominating in this regime

$$V(\varphi) \simeq -ca(\varphi)e^{\delta\varphi}$$

- Logarithmic profiles for scalar field and metric prefactor:

$$\varphi(y) \simeq -\frac{2}{\delta}\log y \qquad \sigma(y) \simeq \pm\frac{4}{(d-2)\delta^2}\log y$$

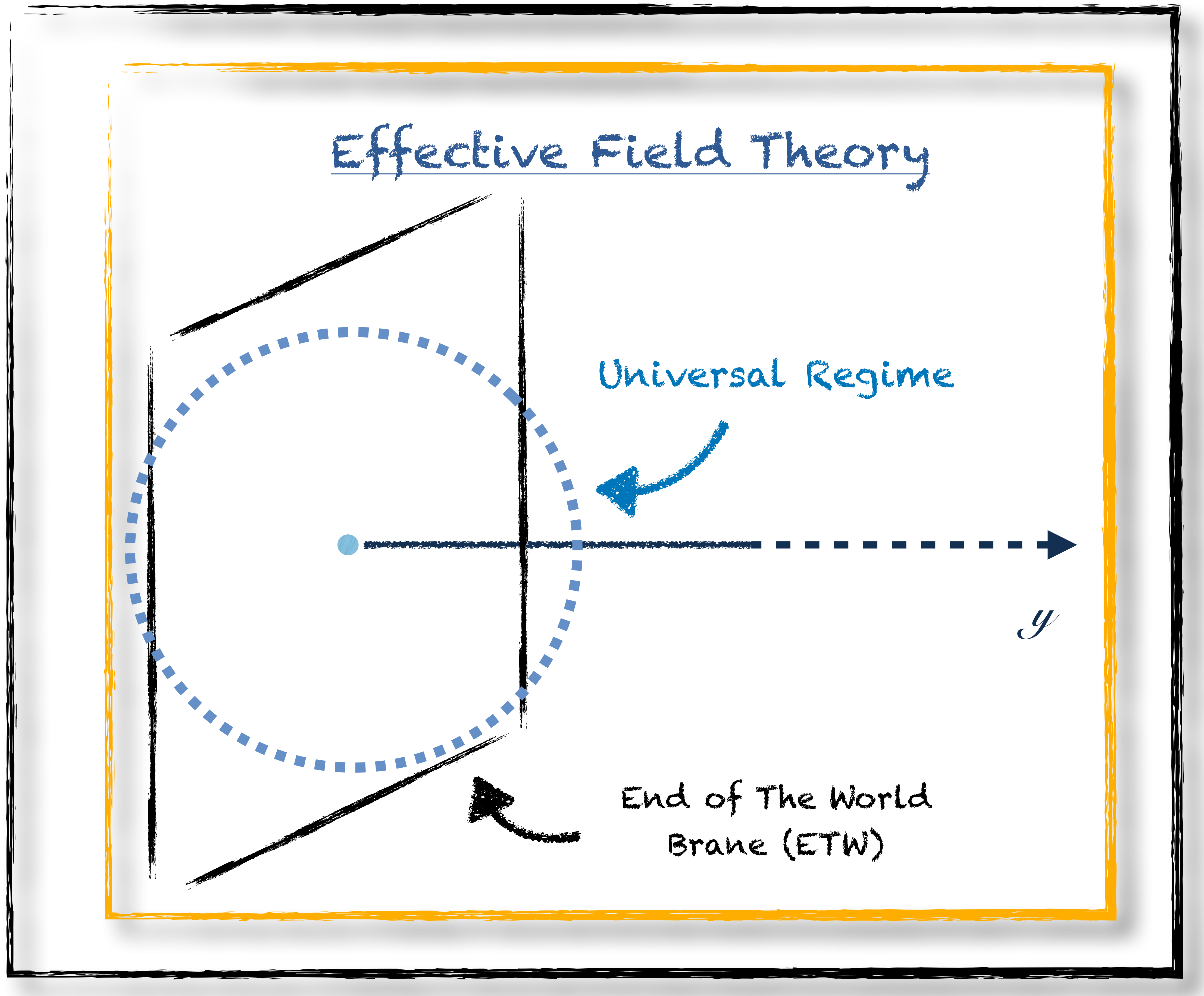
- Scaling relations

$$\Delta \sim e^{-\frac{\delta}{2}\mathscr{D}} \qquad |R| \sim e^{\delta\mathscr{D}}$$

Limits of the Analysis

- Universal behavior only LOCALLY.

- It doesn't provide a microscopic description of the defect.



10d Massive type IIA

Let's consider the 10d massive type IIA theory:

$$S_{10,E} = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\{R - (\partial\phi)^2 - \frac{1}{2}e^{\frac{5}{2}\sqrt{2}\phi}F_0^2 - \frac{1}{2}e^{\frac{\sqrt{2}}{2}\phi}|F_4|^2\}$$

where F_0 denotes the Romans mass parameter.

The Tadpole potential assures us we are dealing with a 9d Poincarè invariant running solution for the EoMs along a spatial coordinate.

Matching the potential of the action with the exponential form of the potential obtained in the Local Analysis we get the value of the critical exponent:

$$V = \frac{1}{2}e^{\frac{5}{\sqrt{2}}\phi}F_0^2 = -ace^{\delta\phi} \quad \longrightarrow \quad \delta = \frac{5}{\sqrt{2}}$$

10d Massive type IIA

Once obtained this exponent the Local Analysis gives us:

- The profile for the dilaton:

$$\phi \simeq -\frac{2\sqrt{2}}{5}\log y$$

- The profile for the function $\sigma(y)$, from which we can determine the metric:

$$\sigma \simeq -\frac{1}{25}\log y \longrightarrow ds^2 \simeq y^{2/25} ds_9^2 + dy^2$$

- The scaling relations:

$$\Delta \sim e^{-\frac{5}{2\sqrt{2}}\mathscr{D}} \qquad |R| \sim e^{\frac{5}{\sqrt{2}}\mathscr{D}}$$

- The scaling relations obtained from the local analysis are in agreement with the scaling relations obtained from the complete solutions:

$$ds_{10}^2 = [-F_0 x^9]^{1/12}\eta_{\mu\nu}dx^\mu dx^\nu$$
$$e^{\sqrt{2}\phi} = [-F_0 x^9]^{-5/6}$$

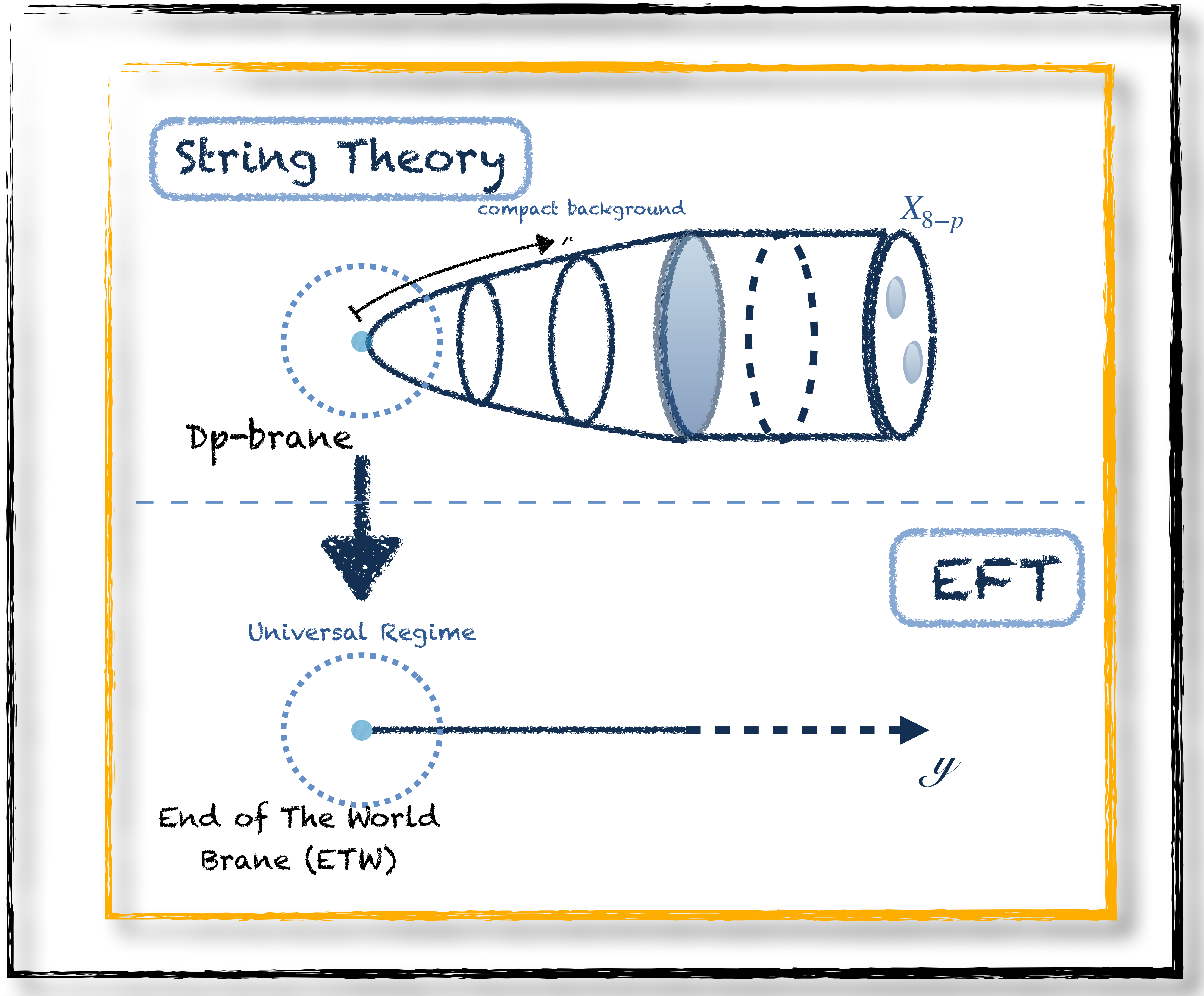
[Buratti, Delgado,Uranga, '21]

- The relation between the complete solutions and those given by the local analysis is done via the following change of coordinates:

$$y = \int_0^{x^9} [-F_0 \tilde{x}^9]^{1/24} d\tilde{x}^9$$

We want to interpret the Dp-branes solution as a Dynamical Cobordism to Nothing associated to compactification with flux.

- Dp-branes are charged objects under RR *(8-p)*-forms.
- Dynamical Cobordisms involving compactifications with flux require the introduction of charged object to absorb the flux and avoid global symmetries



D-branes as Dynamical Cobordisms

Let's consider the following type II action with a dilaton and a RR field:

$$S \sim \frac{1}{2}\int d^{10}x\{R-(\partial\phi)^2-\frac{1}{2n!}e^{a\phi}|F_n|^2\}$$

(with $n=8-p$)

The solutions of the EoMs associated to the action are the *Dp-brane* solutions.

$$ds^2_{10}=\underbrace{Z(r)^{\frac{p-7}{8}}\eta_{\mu\nu}}_{\text{Longitudinal}}+\underbrace{Z(r)^{\frac{p+1}{8}}(dr^2+r^2d\Omega^2_{8-p})}_{\text{Transverse}}$$

$$\phi=\frac{(3-p)}{4\sqrt{2}}\log Z(r)$$

Where the warp factor is given by the *Harmonic functions*:

$$\begin{cases} Z(r)=1+\left(\frac{\rho}{r}\right)^{7-p} & 0\le p\le 6, \quad p\neq 3 \\ Z(r)=1-\frac{N}{2\pi}\log\left(\frac{r}{\rho}\right) & p=7 \end{cases}$$

D-branes structure in terms of the Local description

- S^{8-p} truncation of the Dp-brane solutions

Lower *d-dimensional Einstein*-theory coupled to two scalars with a non-trivial potential arising from the curvature of S^{8-p} and from the flux term.

$$ds_d^2 = \underbrace{\left(\frac{r^2}{r_0^2} Z(r)^{\frac{p+1}{8}}\right)^{\frac{8-p}{p}}}_{\text{Radion}} \{Z(r)^{\frac{p-7}{8}} \eta_{\mu\nu} dx^\mu dx^\nu + Z(r)^{\frac{p+1}{8}} dr^2\} = e^{-2\sigma(r)} ds_{d-1}^2 + Z(r)^{\frac{p+1}{p}} dr^2$$

- I put the metric in the domain-wall form using the change of coordinate:

$$dy = Z(r)^{\frac{p+1}{2p}} \left(\frac{r}{r_0}\right)^{\frac{8-p}{p}} dr$$

- Comparing the profile of $\sigma(y)$ with the result given by the LA we get the critical exponent:

$$\delta^2 = \frac{4(p-3)^2}{p(9-p)}$$

Results from the Local Analysis

$$\mathcal{D}(y) \sim -\frac{\sqrt{p(9-p)}}{|p-3|} \log y \qquad \Delta \sim e^{-\frac{|p-3|}{p(9-p)}\mathcal{D}} \qquad |R| \sim e^{2\frac{|p-3|}{\sqrt{p(9-p)}}\mathcal{D}}$$

- I provided the conditions to identify a Cobordism to Nothing following an Effective approach;
- I wrote an Universal description of the effective theory realizing Cobordism in the regime near the End of The World brane;
- I presented two examples satisfing these conditions and reproducing the Local Analysis.

Outlooks

Study of realizations of **Dynamical Cobordisms** in **Non-critical** String theories with **Tachyons**.

[work in progress with **A.Uranga** and **M.Delgado**]

Thank you!